

Plane Geometry

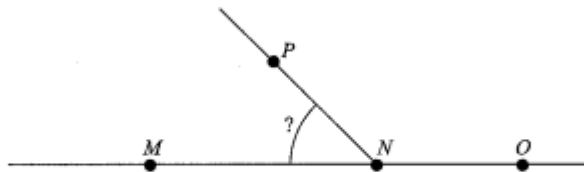
Name _____

DIFFICULTY LEVEL: EASY

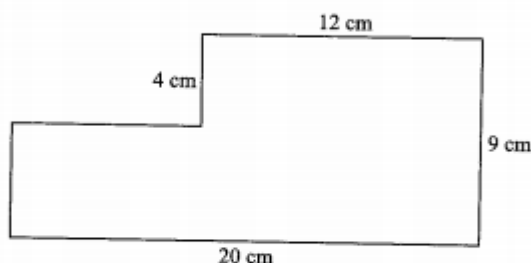
Do your figuring here

- What is the volume, in cubic inches, of a cube whose edges each measure 3 inches in length?
A. 9
B. 12
C. 18
D. 27
E. 81

- In the figure below, M , N , and O are colinear, the measure of angle MNP is $3x^\circ$, and the measure of angle ONP is $6x^\circ$. What is the measure of angle MNP ?

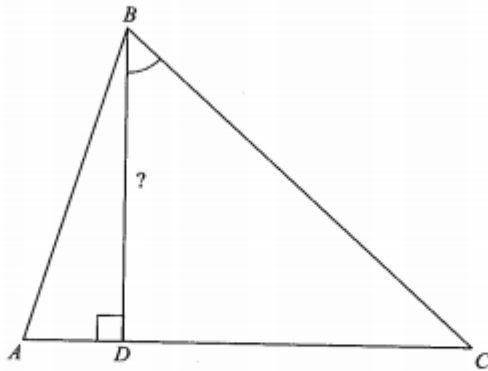


- For the polygon below, the lengths of 2 sides are not given. Each angle between adjacent sides measures 90° . What is the polygon's perimeter, in centimeters?



- 45
- 58
- 87
- 90
- 180

4. The area of $\triangle ABC$ below is 40 square inches. If \overline{AC} is 10 inches long, how long is the altitude \overline{BD} , in inches?

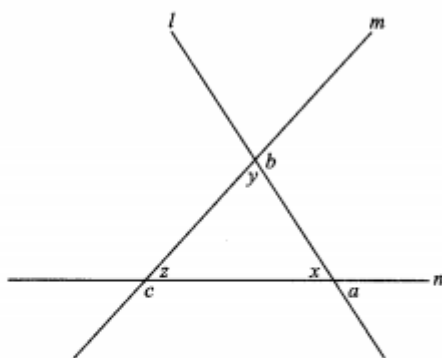


- F. 4
G. 6
H. 8
J. 10
K. 12
5. What is the area, in square inches, of a trapezoid with a height of 6 inches and parallel bases of 9 inches and 7 inches, respectively?
- A. 24
B. 32
C. 48
D. 96
E. 378

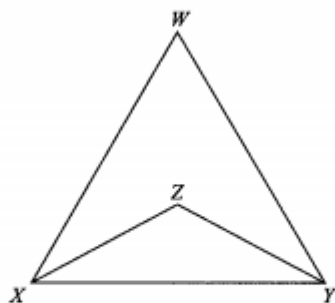
DIFFICULTY LEVEL: MEDIUM

6. The area of a wheel is 78.5 inches. About how many revolutions does one of these wheels make traveling 100 feet (1,200 inches) without slipping?
- F. 12
G. 15
H. 38
J. 100
K. 942

7. In the figure below, if $a = 140$, what is the value of $b + c$?

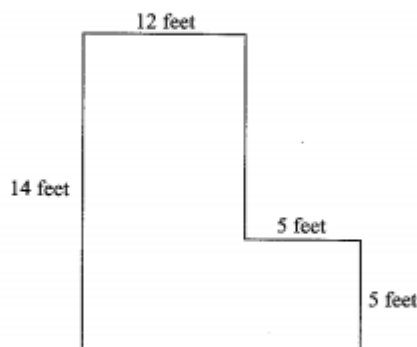


- A. 320°
 B. 220°
 C. 180°
 D. 140°
 E. 50°
8. Triangles WXY and ZXY , shown below, are isosceles with base \overline{XY} . Segments \overline{XZ} and \overline{YZ} bisect $\angle WXY$ and $\angle WYX$, respectively. Which of the following angle congruences is necessarily true?



- F. $\angle WXY \cong \angle WYZ$
 G. $\angle WXZ \cong \angle WYX$
 H. $\angle WXZ \cong \angle XYZ$
 J. $\angle WYZ \cong \angle XWY$
 K. $\angle XYZ \cong \angle XWY$

9. Mandy plans to carpet the entire floor of her bedroom. The floor is flat and all adjacent sides meet at right angles, as shown below. Mandy can purchase 8-foot \times 12-foot pieces of carpet on sale. What is the minimum number of pieces of carpet that she must purchase in order to carpet her bedroom floor?



- A. 1
B. 2
C. 3
D. 4
E. 5
10. Triangle ABC is similar to triangle XYZ . \overline{AB} is 5 inches long, \overline{BC} is 8 inches long, and \overline{AC} is 3 inches long. If the longest side of $\triangle XYZ$ is 20 inches long, what is the perimeter, in inches, of $\triangle XYZ$?
- F. 16
G. 28
H. 40
J. 64
K. 88

DIFFICULTY LEVEL: HARD

Do your figuring here

11. The noncommon rays of 2 adjacent angles form a straight angle. The measure of one angle is 3 times the measure of the other angle. What is the measure of the smaller angle?
- A. 40°
B. 45°
C. 50°
D. 55°
E. 60°

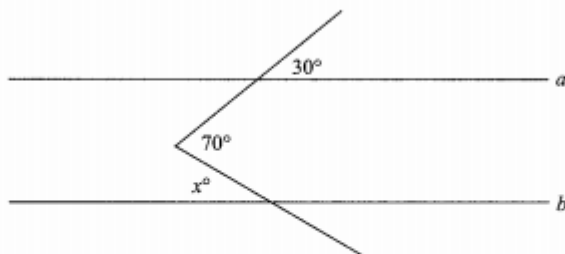
12. A square has sides that are the same length as the radius of a circle. If the circle has a circumference of 64π square units, how many units long is the perimeter of the square?

F. 8
G. 16
H. 32
J. 128
K. 256

13. In a certain rectangle, $PQRS$, angle QPS and angle PSR are right angles. If the length of line \overline{PR} is 34 units and the length of line \overline{PS} is 30 units, what is the length of line \overline{RS} ?

A. $\sqrt{30}$
B. 16
C. $\sqrt{34}$
D. $2\sqrt{514}$
E. 14

14. In the figure below, lines a and b are parallel and angle measures are as marked. If it can be determined, what is the value of x ?



F. 30°
G. 40°
H. 55°
J. 70°
K. Cannot be determined from the given information.

15. Which of the following degree measures is equivalent to 3.75π radians?

A. $2,700^\circ$
B. $1,350^\circ$
C. 675°
D. 337.5°
E. 225°

16. The radius of a circle is $\frac{32}{\pi}$ centimeters. What is the area of the circle?

F. 64
G. 32π
H. $\frac{1,024}{\pi}$
J. 1,024
K. $1,024\pi$

PLANE GEOMETRY—DIFFICULTY LEVEL: EASY

1. **The correct answer is D.** The volume of a rectangular prism is $l \times w \times h$. A cube is a special case in which the length, width, and height are all equal. The volume of a cube with edges of length 3 is $3 \times 3 \times 3 = 3^3 = 27$.

2. **The correct answer is H.** Because points M , N , and O are colinear, angle MNO has a measure of 180° . When split into two component angles, the sum of those component angles must equal 180° . Therefore the measure of angle MNP plus the measure of angle ONP equals 180° . Since the measure of angle MNP is $3x^\circ$, and the measure of angle ONP is $6x^\circ$, it follows that $3x + 6x = 180$:

$$\begin{aligned} 3x + 6x &= 180 \\ 9x &= 180 \\ x &= 20 \end{aligned}$$

The measure of angle MNP is $3x^\circ = 3(20)^\circ = 60^\circ$

3. **The correct answer is B.** Because all of the angles are known to be right angles, you can conclude that the length of all the right-facing sides must equal the length of all the left-facing sides. Since the length of the right-facing side is 9, the missing left-facing side will have length 5. Similarly, the length of all up-facing sides must equal the length of all down-facing sides, making the length of the missing up-facing side 8. The perimeter (beginning with the left-facing side and moving clockwise) is $5 + 8 + 4 + 12 + 9 + 20 = 58$ cm.

4. **The correct answer is H.** The area of a triangle is $\frac{1}{2}bh$, where b is the length of the base and h is the height of the triangle. In this case, \overline{AC} is the base of the triangle and \overline{BD} is the height. Substitute the given values into the formula and solve:

$$40 = \frac{1}{2}(10)h$$

$$40 = 5h$$

$$8 = h; \overline{BD} \text{ has length } 8.$$

5. **The correct answer is C.** To solve this problem, recall that the area of a trapezoid is found by multiplying the average of the parallel bases by the height. Since the parallel bases have length 9 and 7, the average is $\frac{(9+7)}{2} = \frac{16}{2} = 8$. The area is the average of the bases multiplied by the height, or $8 \times 6 = 48$ square inches.

PLANE GEOMETRY—DIFFICULTY LEVEL: MEDIUM

6. **The correct answer is H.** When a wheel makes one revolution, it goes completely around one time. The distance one time around a wheel is equal to the wheel's circumference. A wheel is a circle, so the formula for the circumference of a wheel is $C = 2\pi r$. You are given that the area of the wheel is 78.5 inches. The formula for the area of a circle is $A = \pi r^2$, so $78.5 = \pi r^2$. Solve for r , the radius, as follows:

$$78.5 = \pi r^2$$

$$78.5 = 3.14(r^2)$$

$$25 = r^2$$

$$5 = r$$

Now calculate the circumference, C , of the wheel, as follows:

$$C = 2\pi(5)$$

$$C = 2(3.14)(5) = 31.4$$

Because the circumference of the wheel is 31.4, one revolution of the wheel is equal to 31.4 inches. Divide the total number of inches traveled (1,200) by 31.4 to find the number of revolutions the wheel makes:

$$1,200 \div 31.4 = 38.2; \text{ the wheel makes about 38 revolutions.}$$

7. **The correct answer is A.** To solve this problem, use the fact that supplementary angles add up to 180° . In the figure shown, x , y , and z form relationships with a , b , and c , respectively. Because a and x are vertical angles, they have the same measure.

Angles b and y are supplementary, so they add up to 180, making $y = 180 - b$. Likewise, $z = 180 - c$. Therefore, the sum of the angles within the triangle is $180 = x + y + z$, which is equivalent to $a + (180 - b) + (180 - c)$. You are given that $a = 140$, so substitute 140 for a , as follows:

$$180 = 140 + (180 - b) + (180 - c)$$

$$180 = 140 + 180 + 180 - b - c$$

$$180 = 500 - b - c$$

$$-320 = -b - c$$

$$320 = b + c$$

8. The correct answer is H. Because you are given that segments \overline{XZ} and \overline{YZ} bisect $\angle WXY$ and $\angle WYX$, respectively, you can conclude the following:

$$\begin{aligned}\angle WXZ &\cong \angle YXZ \\ \angle WYZ &\cong \angle XYZ\end{aligned}$$

Furthermore, because you know that these two triangles are isosceles, you know that the base angles are congruent. Because of this congruence, you can conclude that the four angles written above are all congruent: $\angle WXZ \cong \angle YXZ \cong \angle WYZ \cong \angle XYZ$. Therefore, through transitivity, you can conclude that $\angle WXZ \cong \angle XYZ$.

9. The correct answer is C. Since Mandy's room is an L-shape comprised of a 14-foot \times 7-foot rectangle and a 5-foot \times 5-foot square, the area of her room is $(14 \times 7) + (5 \times 5) = 98 + 25$, or 123 square feet. An 8-foot \times 12-foot piece of carpet covers 96 square feet ($8 \times 12 = 96$), so the area of 123 square feet is slightly larger than what 1 piece of carpet will cover. Therefore, Mandy will need at least 3 pieces of carpet.

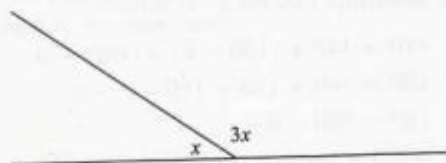
10. The correct answer is H. When \overline{AB} is 5 inches long, \overline{BC} is 8 inches long, and \overline{AC} is 3 inches long, $\triangle ABC$ has a perimeter of $5 + 8 + 3 = 16$. To find the perimeter of $\triangle XYZ$ when its longest side is 20, set up proportions to find the lengths of the sides. The sides of similar triangles are in proportion to each other. In this case, you can take the proportion of the longest sides, $20:8 \left(\frac{20}{8}\right)$, and apply it to the perimeter, p , as follows:

$$\begin{aligned}\frac{20}{8} &= \frac{p}{16} \\ p &= \frac{(20 \times 16)}{8} = 40\end{aligned}$$

PLANE GEOMETRY—DIFFICULTY LEVEL: HARD

11. The correct answer is B. To solve this problem, remember that the noncommon rays of two adjacent

angles are the sides of the angles that are *not* shared. As shown in the figure below, these rays form a straight angle, or straight line, which you know contains 180° .

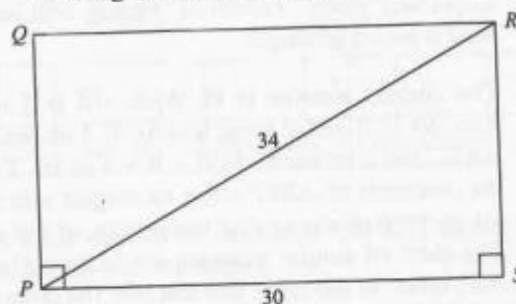


To find the measure of either angle, set up an equation: x (the measure of the smaller angle) + $3x$ (the measure of the larger angle) = 180° . Solve for x :

$$\begin{aligned}4x &= 180^\circ \\ x &= 45^\circ\end{aligned}$$

12. The correct answer is J. The circumference of a circle is given by $2\pi r$, where r is the radius. If a circle has a circumference of 64π square units, then $64\pi = 2\pi r$. Dividing both sides by 2π yields $r = 32$. Since the square has sides that are the same length as the radius of the circle, 32, the perimeter of that square is $4(32) = 128$.

13. The correct answer is B. You are given that angle $\angle QPR$ and angle $\angle PRS$ are right angles; you are also given the lengths of diagonal \overline{PR} (34), and side \overline{PS} (30). It may help to write the lengths of the various line segments on a diagram as shown below:



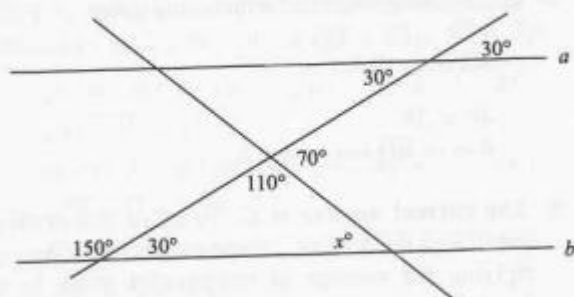
You should now see that you have the length of one side of the right triangle $\triangle PRS$ (30), and the length of the hypotenuse (34). Use the Pythagorean theorem to calculate the length of the remaining side:

$$\begin{aligned}a^2 + b^2 &= c^2 \\ 30^2 + b^2 &= 34^2 \\ 900 + b^2 &= 1,156 \\ b^2 &= 256 \\ b &= 16\end{aligned}$$

The length of \overline{RS} is 16.

NOTE: Solution for #9 has the correct solution but the explanation has inaccurate details such as 14-foot \times 7-foot should be 14-foot \times 12-foot.

14. The correct answer is G. To solve this problem, it is helpful to extend the lines of the angle given in the diagram as shown below:



Make use of the properties of angles made by transversals that cross the parallel lines a and b . Also note the newly created triangle, which angle x is a part of. Given that opposite angles are congruent, one angle of that triangle is equal to 30° . Additionally, the supplement of the 70° is 110° . Knowing that two angles of a triangle are 110° and 30° , $x^\circ + 140^\circ = 180^\circ$, and $x = 40^\circ$.

15. The correct answer is C. To solve this problem, use the fact that π radians is equal to 180° . Given a radian measure, to convert to degrees simply divide by π and multiply by 180:

$$\frac{3.75\pi}{\pi} = 3.75$$

$$3.75 \times 180 = 675^\circ$$

16. The correct answer is H. The area of a circle is given by πr^2 , where r is the radius. If the radius is $\frac{32}{\pi}$, then the area is $\pi \left(\frac{32}{\pi}\right)^2 = \frac{1,024\pi}{\pi^2} = \frac{1,024}{\pi}$.